Derivation of causal mediated effect in pretest-posttest control group design

Valente, M. & MacKinnon, D. P. (2017). Comparing models of change to estimate the mediated effect in the pretest-posttest control group design. *Structural Equation Modeling: A Multidisciplinary Journal, 24* (3) 428-450.

Using the potential outcomes framework, the causal estimator of the mediated effect in the two-wave model is equivalent to *am2xby2m2* in ANCOVA when there is no *XM* interaction present and reduces to the equations used for defining the causal effects for the single mediator model conditional on baseline covariates (VanderWeele & Vansteelandt, 2009). This appendix shows the steps to compute the causal mediated effect and causal direct effect in the pretest-posttest control group design.

Equations A1 and A2 display the conditional expected value of the mediator at posttest and the outcome at posttest, respectively.

|  |  |
| --- | --- |
|  | (A1) |
|  | (A2) |

Define Nested Potential Outcome

|  |  |
| --- | --- |
|  | (A3.1) |
|  | (A3.2) |

Integrate over *M2* and Expand terms

|  |  |
| --- | --- |
|  | (A3.3) |
|  | (A3.4) |

Define remaining Nested Potential Outcomes

|  |  |
| --- | --- |
|  | (A4.1) |
|  | (A4.2) |
|  | (A4.3) |
|  | (A4.4) |

|  |  |
| --- | --- |
|  | (A5.1) |
|  | (A5.2) |
|  | (A5.3) |
|  | (A5.4) |

|  |  |
| --- | --- |
|  | (A6.1) |
|  | (A6.2) |
|  | (A6.3) |
|  | (A6.4) |

Define Indirect Effects as differences between nested potential outcomes.

Total Natural Indirect Effect (TNIE)

|  |  |
| --- | --- |
|  | (A7.1) |
|  | (A7.2) |

Constants not involving *x* or *x\** drop out

|  |  |
| --- | --- |
|  | (A7.3) |

Pure Natural Indirect Effect (PNIE)

|  |  |
| --- | --- |
|  | (A8.1) |
|  | (A8.2) |

Constants not involving *x* or *x\** drop out

|  |  |
| --- | --- |
|  | (A8.3) |

Define Direct Effects as differences between nested potential outcomes.

Total Natural Direct Effect (TNDE)

|  |  |
| --- | --- |
|  | (A9.1) |
|  | (A9.2) |

Constants not involving *x* or *x\** drop out

|  |  |
| --- | --- |
|  | (A9.3) |

Pure Natural Direct Effect (PNDE)

|  |  |
| --- | --- |
|  | (A10.1) |
|  | (A10.2) |

Constants not involving *x* or *x\** drop out

|  |  |
| --- | --- |
|  | (A10.3) |

VanderWeele, T., & Vansteelandt, S. (2009). Conceptual issues concerning mediation, interventions and composition. *Statistics and its Interface*, *2*(4), 457-468.

Derivation of differences between *b* path estimates

Valente, M. & MacKinnon, D. P. (2017). Comparing models of change to estimate the mediated effect in the pretest-posttest control group design. *Structural Equation Modeling: A Multidisciplinary Journal, 24* (3) 428-450.

Because the focus of this study is on randomized pretest-posttest control group design, the unstandardized *a* path is equivalent across the models in expectation assuming successful randomization of units to treatment and control groups. The focus of this analytical comparison of path coefficients of the mediated effect will focus on the *b* path of the mediated effect.

Difference in *b* path across ANCOVA and Difference Score Model expressed as an unmeasured confounder problem. Equations B1-B5.1 express the full difference score model for and and the residual of prediction from the difference score model that is applied in the article.

|  |  |
| --- | --- |
|  | (B1) |
|  | (B2) |
|  | (B3) |
|  | (B4) |
|  | (B5.1) |

For simplicity, all covariance terms involving *X* and *M1* and *X* and *Y1* are not included because they are equal to zero assuming successful randomization.

|  |  |
| --- | --- |
|  | (B5.2) |
|  | (B6.1) |
|  | (B6.2) |

Results from Clark (2005) and Hanushek and Jackson (1977) can be applied to use the variance-covariance information from above to define the difference in the *b* path across models as a single unstandardized regression coefficient. Assuming the relation of the residual to the observed is equal to one and the covariance between *X* and the residual is zero, formula B7 is the difference in unstandardized regression coefficients across the ANCOVA and difference score models.

|  |  |
| --- | --- |
|  | (B7) |
|  | (B8) |

Therefore, when the term is equal to zero, the unstandardized *b* path in the difference score model (will equal the unstandardized *b* path in the ANCOVA model ().

Difference in *b* path across ANCOVA and Residualized Change Score Model. Equations B9-B13.1 express the full residualized change score model for and and the residual of prediction from the residualized change score model that is applied in the article.

|  |  |
| --- | --- |
|  | (B9) |
|  | (B10) |
|  | (B11) |
|  | (B12) |
|  | (B13.1) |

For simplicity, all covariance terms involving *X* and *M1* and *X* and *Y1* are not included because they are equal to zero assuming successful randomization.

|  |  |
| --- | --- |
|  | (B13.2) |
|  | (B14) |
|  | (B15) |

We can use the variance-covariance information from above to define the difference in the *b* path across models as a single regression coefficient.

|  |  |
| --- | --- |
|  | (B16) |
|  | (B17) |

Therefore, when the term is equal to zero, the unstandardized *b* path in the residualized change score model (will equal the unstandardized *b* path in the ANCOVA model ().

Difference in *b* path across ANCOVA and Cross-sectional Model. Equations B18-B21.1 express the full cross-sectional model for and and the residual of prediction from the cross-sectional model that is applied in the article.

|  |  |
| --- | --- |
|  | (B18) |
|  | (B19) |
|  | (B20) |
|  | (B21) |
|  | (B22.1) |

For simplicity, all covariance terms involving *X* and *M1* and *X* and *Y1* are not included because they are equal to zero assuming successful randomization.

|  |  |
| --- | --- |
|  | (B22.2) |
|  | (B23) |
|  | (B24) |

We can use the variance-covariance information from above to define the difference in the *b* path across models as a single regression coefficient.

|  |  |
| --- | --- |
|  | (B25) |
|  | (B26) |

Therefore, when the term is equal to zero, the unstandardized *b* path in the cross-sectional model (will equal the unstandardized *b* path in the ANCOVA model ().

Clark, K. A. (2005). The phantom menace: Omitted variable bias in econometric research. *Conflict Management and Peace Science*, *22*, 341-353.

Hanushek, E. A., & Jackson, J. E. (1977). *Statistical methods for the social scientists*. New York, NY: Academic Press.